

Flood Development Process Forecasting Based on Water Resources Statistical Data

Oleg Mandryk¹, Andriy Oliynyk², Roman Mykhailiuk³, Lidiia Feshanych^{*4}

¹Department of Ecology, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine.
Email: o.mandryk@nung.edu.ua

²Department of Applied Mathematics, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine. Email: andrioliiny@gmail.com

³Department of Ecology, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine.
Email: Mromanm2016@gmail.com

⁴Department of Automation and Computer Integrated Technologies, Ivano-Frankivsk National Technical University of Oil and Gas, Ivano-Frankivsk, Ukraine. Email: lidiia.feshanych@gmail.com

**Corresponding Author | ORCID: 0000-0002-5156-2199*

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Abstract

The Ukrainian Carpathians is the territory with a great threat of floods. This is due to natural and climatic conditions of this region, which is characterized by mountainous terrain, high density of hydrological network and a significant amount of precipitation. Amount of precipitation here ranges from 600 mm on plains to 1,600 mm on mountain tops. The main factors of floods occurrence are excessive precipitation, low water permeability of soil and a high proportion of low-permeability rocks (flysch layers with a predominance of clay layers). Therefore, catastrophic floods in the region were also observed in previous centuries, when the anthropogenic impact on the environment, including forest ecosystems, was not comparable with the current one. Any flood is characterized by a period of development, a period of its critical (maximum) intensity and a period of decline. In the present paper, based on the use of methods for approximating the curves and the results of experimental studies of flood waters, a method of mathematical description and forecasting of the flood development is suggested. The recommended direction of further research may be related to the development of experimental means to determine the parameters that affect the process of flood formation.

Keywords

Floods; Forecasting; Mathematical description; Water resources



Introduction

The Ukrainian Carpathians is the territory with a great threat of floods. This is due to natural and climatic conditions of this region characterized by mountainous terrain, high density of hydrological network and a significant amount of precipitation. Amount of precipitation here ranges from 600 mm on plains to 1600 mm on mountain tops. The main factors of flood occurrence are excessive amount of precipitation, low water permeability of soil and a high proportion of low-permeability rocks (flysch layers with a predominance of clay layers). Therefore, catastrophic floods in the region were also observed in previous centuries, when the anthropogenic impact on the environment, including forest ecosystems, was not comparable with the current one (Figure 1) (Mandryk *et al.*, 2017; Arkhypova *et al.*, 2019).



Figure 1: Flood in Ivano-Frankivsk region (June 2020). Courtesy: Photo from open sources

The rivers of the Carpathian which have flood hazard area belong to transboundary waters. In the basins of Carpathian rivers, some part of which belong to neighboring countries, the European Union Water and Flood Directives are implemented in accordance with the Association Agreement between Ukraine and the European Union. These directives are implemented with the funding from European Union (Prykhodko *et al.*, 2020; Kinash *et al.*, 2019).

The estimation of the level of flood waters is an urgent scientific and technical problem in the context of complex geo-climatic processes, which lead to catastrophic floods, taking place in European countries (such as the floods of Ukraine in 1927, 1941, 1969, 1974, 1980, 1998, 2001, 2008, 2010 and 2020, of Poland and Slovakia in 2012, of the Czech Republic and Great Britain in 2013, and of Germany in 2002). The stated problem is addressed by a wide range of scientists studying and solving the applied problems by using precise models of filtrating flows (Oliynyk and Panchuk, 1992; Oliynyk and Steyer, 2012) and empirical dependences (Mandryk, Pukish and Zelmanovych, 2017; Maslova and Susidko, 2006; Sosedko, 1980; Leontiev, 2009), which are established on the basis of experimental data analysis (Zasidko *et al.*, 2019;

Pietrzak *et al.*, 2018). However, the above-mentioned works do not suggest the idea of the possibility to build the express method for forecasting the phenomenon of flood development.

In the present paper, based on the use of methods for approximating the curves and the results of experimental studies of flood waters, a method of mathematical description and forecasting of the flood development is suggested. Presented method gives next possibility:

- to apply the mathematical methods for studying of real processes (floods, seasonal phenomena of emergency water distribution;
- to offer the functional analytical structure based on the statistical information about the floods in concrete region, which give the possibility to predict the flood's duration and intensity;
- to restore the ordinary differential equation which describe the flood process development using the information about one's solution;
- to choose the factors that have the significant influence on the flood's characteristics; and

Methodology

Mathematical modeling of the flood development process

When analyzing the real phenomena of flood situations, it is possible to depict schematically the relationship between time (t) and the level of flood waters (l) (Figure 2).

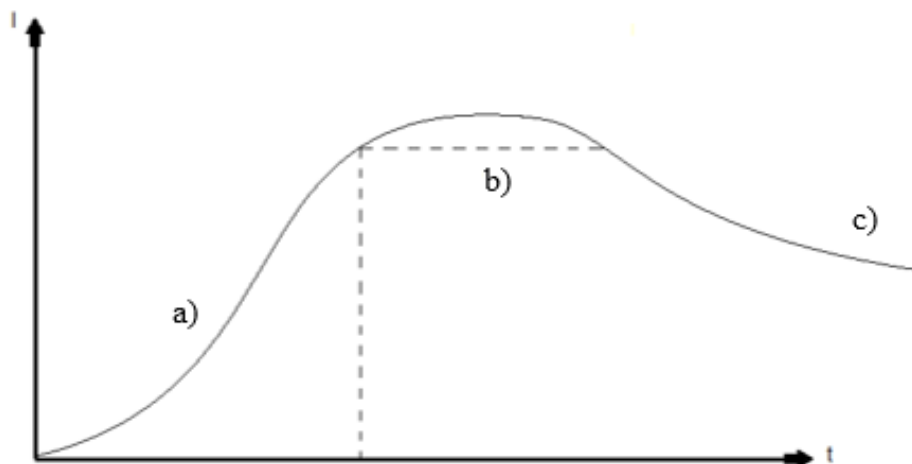


Figure 2: Diagram of the relationship between time and the level of flood waters

Any flood is characterized by a period of development a), a period of its critical (maximum) intensity b) and a period of decline c). The scheme shown in figure 2 does not consider cases when for a short period of time the flood has several peaks. In this case the phenomenon shown in Figure 2 can be simulated by application of several floods with one peak. It is necessary to reproduce the analytical structure of the function shown in Figure 2. From a mathematical point of view, this function must be characterized by the following conditions:

$$\begin{cases} f(0) = 0; \\ \exists x_0 : f'(0) = 0; \quad f''(x_0) < 0; \\ \lim_{x \rightarrow \infty} f(x) = 0. \end{cases} \quad (1)$$

Obviously, many functions meet such conditions, and, therefore, the problem of constructing the flood development model from a mathematical point of view is ill-defined (Leontiev, 2009) to regulate. It is

needed to develop certain algorithms using additional information about type of the function that would satisfy the conditions (1) (Tikhonov and Arsenin, 1979; Zorich, 1981; Filippov, 2007).

The simplest of the known functions is the following one:

$$f(t) = t \cdot e^{-t}, \quad (2)$$

the graph of which is shown in Figure 3.

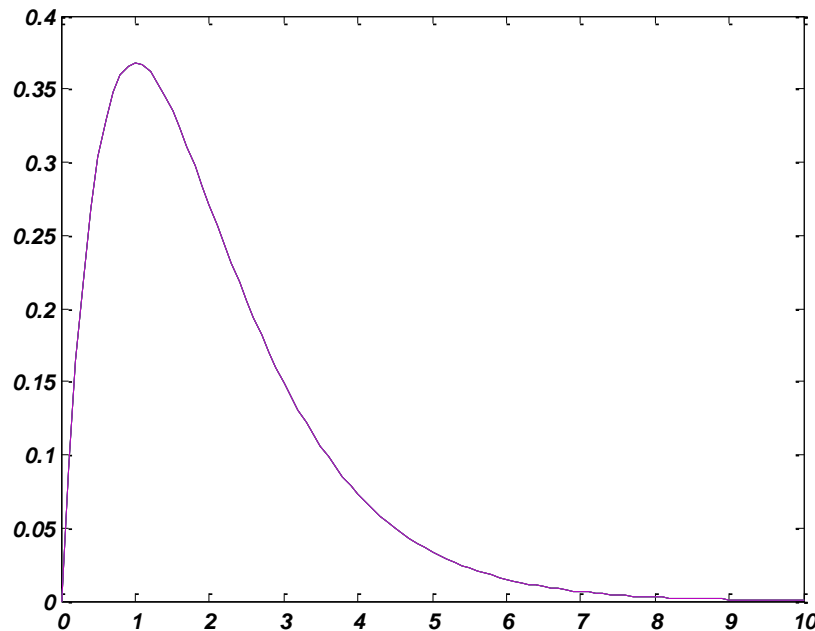


Figure 3: The graph of the function $f(t) = t \cdot e^{-t}$

Verified by means of direct check, it has been established that, for the function (2), all three following conditions (1) are met:

1. $f(0) = 0 \cdot e^{-0} = 0$;
2. $\exists f(x_0) : f'(x_0) = 0$;
 $(t \cdot e^{-t})' = e^{-t} - t \cdot e^{-t} = 0 \rightarrow t = 1$;
 $f''(t) = -e^{-t} - e^{-t} + t \cdot e^{-t} = (-2+t) \cdot e^{-t}$;
 $f''(1) = -e^{-1} < 0$;
3. $\lim_{t \rightarrow \infty} \frac{t}{e^t} = \lim_{t \rightarrow \infty} \frac{1}{e^t} = 0$.

However, application of function (2) to describe flood phenomena is connected with some problems. It cannot be changed, and its type cannot be selected according to certain results of experimental studies and analysis of statistical data on floods. To solve this problem, the following function can be suggested:

$$y = f(t) = t \cdot e^{-at}, \quad a > 0, \quad (3)$$

which obviously also satisfies the stated conditions (Figure 4):

1. $f(0) = 0 \cdot e^{-a \cdot 0} = 0$;
2. $\exists f(x_0) : f'(x_0) = 0$;
 $(t \cdot e^{-at})' = e^{-at} - a \cdot t \cdot e^{-at} = (1 - a \cdot t) \cdot e^{-at} = 0$;
 $t = \frac{1}{a}$;

$$f''(t) = -a \cdot e^{-at} + (1 - a \cdot t) \cdot (-a) \cdot e^{-at} = e^{-at} \cdot (-2 \cdot a + a^2 \cdot t) = 0;$$

$$f''\left(\frac{1}{a}\right) = -e^{-1} \cdot (-2 \cdot a + a) = -a e^{-1} < 0;$$

$$3. \quad \lim_{t \rightarrow \infty} \frac{t}{e^{at}} = \lim_{t \rightarrow \infty} \frac{1}{a \cdot e^{at}} = 0.$$

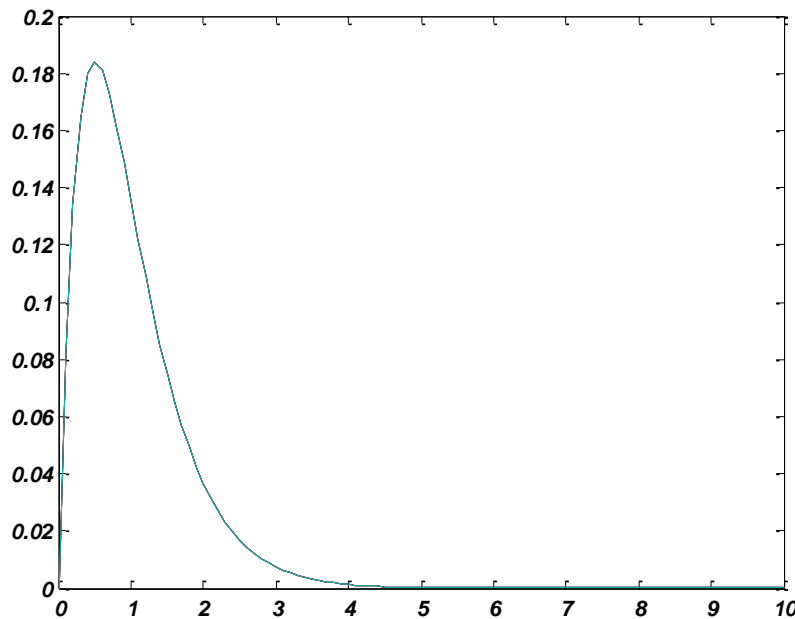


Figure 4: The graph of the function $y = f(t) = t \cdot e^{-at}$, $a > 0$

Dependence (3) has a parameter $a > 0$ that allows to obtain a whole range of curves, which, however, are topologically (according to their spatial location) similar, and that does not allow to introduce many parameters on which the level of flood water rising depends. That is why, the following two-parameter model is proposed to determine the function of type (1) of the form (Figure 5).

$$y = t^n \cdot e^{-at}, \quad a > 0, n > 0, \quad (4)$$

for which all conditions (1) are determined:

$$1. \quad (t^n \cdot e^{-at})' = n \cdot t^{n-1} \cdot e^{-at} - a \cdot t^n \cdot e^{-at} = t^{n-1} \cdot (n \cdot e^{-at} + a \cdot t \cdot e^{-at}) = t^{n-1} \cdot e^{-at} \cdot (n - a \cdot t) = 0 \rightarrow t = \frac{n}{a};$$

$$\begin{aligned} (t^n \cdot e^{-at})'' &= -a \cdot e^{-at} \cdot t^{n-1} \cdot (n - a \cdot t) + e^{-at} \cdot t^{n-2} \cdot (n-1) \cdot (n - a \cdot t) + e^{-at} \cdot t^{n-1} \cdot (-a) = \\ &= e^{-at} \cdot t^{n-2} \cdot [(-a) \cdot t \cdot (n - a \cdot t) + (n-1) \cdot (n - a \cdot t) - a \cdot t] = e^{-at} \cdot t^{n-2} \cdot [-a \cdot t \cdot n + a^2 \cdot t^2] + \\ &+ (n^2 - n - n \cdot a \cdot t + a \cdot t) - a \cdot t = e^{-at} \cdot t^{n-2} \cdot [-a \cdot t \cdot n + a^2 \cdot t^2 + n^2 - n - n \cdot a \cdot t] = \\ &= e^{-at} \cdot t^{n-2} \cdot [a^2 \cdot t^2 - 2 \cdot a \cdot t \cdot n + n^2 - n] = e^{-at} \cdot t^{n-2} \cdot [(a \cdot t - n)^2 - n] \end{aligned}$$

$$f''\left(\frac{n}{a}\right) = -e^{-n} \cdot \left(\frac{n}{a}\right)^{n-2} \cdot [(n-n)^2 - n] = -n \cdot e^{-n} \left(\frac{n}{a}\right)^{n-2} < 0;$$

$$2. \quad f(0) = 0;$$

$$3. \quad \lim_{t \rightarrow \infty} \frac{t^n}{e^{at}} = 0 \text{ according to the L'Hospital's Rule (Zasidko et al., 2019).}$$

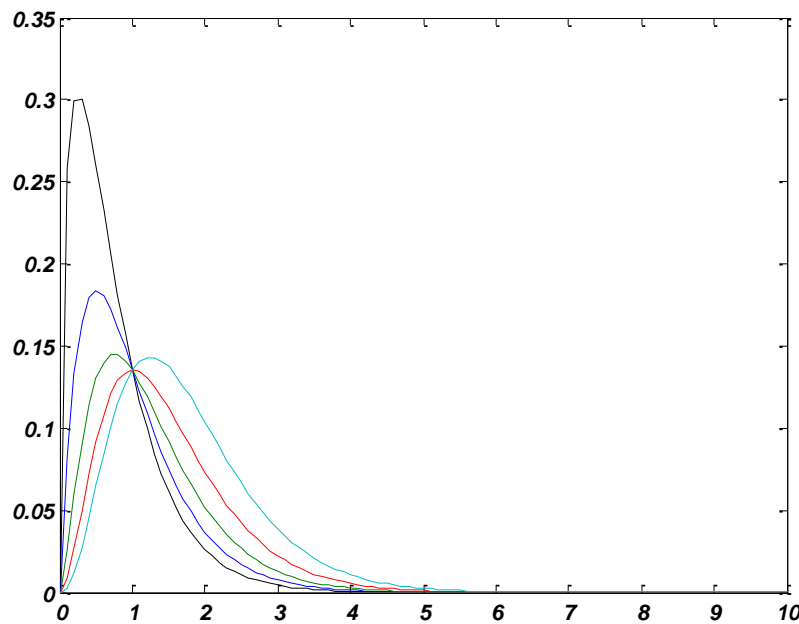


Figure 5: Family of graphs for the function $y = t^n \cdot e^{-at}$, $a > 0$, $n > 0$

By applying, for example, dependence (4), it is possible to restore the differential equation of the process described by each of the functions (2) – (4).

To fulfill this, we can use the following property of linear systems: if the function $y_1(x)$ satisfies the first-order differential equation of the form $f(y, y') = 0$, then the following condition is met (Berestneva, Marukhina and Sheveliov, 2012):

$$\begin{vmatrix} y(x) & y'(x) \\ y_1(x) & y_1'(x) \end{vmatrix} = 0. \quad (5)$$

Then, by substituting in the determinant (5) of the function $y_1(x)$, according to the properties of determinants, the equation (5) is satisfied identically. Applying similar values, it is possible to obtain differential equations, the solutions of which are the corresponding functions: for the function (2), equation (5) requires the form:

$$\begin{vmatrix} y' & y \\ e^{-t} - t \cdot e^{-t} & t \cdot e^{-t} \end{vmatrix} = 0$$

or after summing up the similar addends:

$$y' \cdot t - (1-t) \cdot y = 0, \quad (6)$$

for the function of type (3), the form:

$$\begin{vmatrix} y & y' \\ t \cdot e^{-at} & e^{-at} - t \cdot a \cdot e^{-at} \end{vmatrix} = 0$$

or:

$$y' \cdot t - (1-t) \cdot y = 0, \quad (7)$$

and for the function of type (4), the form:

$$\begin{vmatrix} y & y' \\ t^n \cdot e^{-at} & n \cdot t^{n-1} e^{-at} - t^n \cdot a \cdot e^{-at} \end{vmatrix} = 0,$$

where the following values are obtained:

$$y' \cdot t^n \cdot e^{-at} - t^n \cdot e^{-at} \cdot \left(\frac{n}{t} - a\right) \cdot y = 0 ;$$

$$y' = \left(\frac{n}{t} - a\right) \cdot y . \quad (8)$$

For each of the equations (6) – (8), especially for equation (8), there can be set the initial conditions of the form:

$$y(t_0) = y_0 , \quad (9)$$

setting the conditions in the form of (9) allows to take into account the initial level of flooding:

$$y = y_0 \cdot e^{-at_0} \cdot e^{-at} \cdot t_0^{-n} \cdot t^n = y_0 \cdot \left(\frac{t}{t_0}\right)^n \cdot e^{-a(t-t_0)}$$

It is also important to establish the physical content of coefficients n and a , which may be functions of the form:

$$\begin{cases} n = n(x_1, x_2, \dots, x_k, t); \\ a = a(x_1, x_2, \dots, x_k, t), \end{cases} \quad (10)$$

where the values are soil permeability, air humidity, water intake humidity, relief features, etc., and the variable is time.

The question arises how to determine the numerical values of parameters (10) for different types of floods and how to choose the variables related to (10), which most significantly influence these dependencies. It is also necessary to define the components of the equation (8). Function $y(t)$ represents the level of flood waters at some point of time, which varies in proportion to itself with an alternating-sign coefficient of the form:

$$k = \frac{n}{t} - a , \quad (11)$$

where n and a are empirically determined coefficients. When $k > 0$, the intensity of the flood increases; when $k < 0$, it decreases.

Let the statistical data on its course (y_i, t_i) are known for the chosen flood. According to these values (y_i, t_i) $i=1, \dots, N$, where N is the number of observations, which during intense floods can be quite significant due to the fact that flood water levels are monitored regularly, estimation of parameters n and a can be done by using a linear regression apparatus. For this purpose, function (4) can be written (applying logarithm problem) as:

$$\ln y = n \ln t - at,$$

$$\frac{\ln y}{t} = n \frac{\ln t}{t} - a, \quad (12)$$

introducing the notation $\varphi = \frac{\ln y}{t}$; $\varphi_0 = \frac{\ln t}{t}$; $k = n$; $\beta = -a$, we obtain:

$$\varphi = k\varphi_0 + \beta, \quad (13)$$

that is, the linear regression equation. Using the known formulas (Zamikhovskii *et al.*, 2014) for linear regression coefficients, we obtain $y' \cdot t - (1-t) \cdot y = 0$:

$$k = \frac{N \sum \varphi \varphi_0 - \sum \varphi \sum \varphi_0}{N \sum \varphi_0^2 - (\sum \varphi_0)^2};$$

$$\beta = \frac{1}{N} \left(\sum \varphi_i - k \sum \varphi_0 \right), \quad (14)$$

from where, using communication formulas, is obtained the following:

$$n = k \cdot \beta = -a, \quad (15)$$

that is, parameters n and a can be determined unambiguously, that allows us to talk about the constructed regulable algorithm for the ill-defined problem of restoring the function under conditions (1). To determine the factors influencing the process, the method of (Pietrzak *et al.*, 2018) associative analysis is used, which allows to identify variables and parameters that affect the process of increasing flood waters. Suppose, according to the results of experimental research and analysis of statistical data, a quantitative characteristic of some parameter x_i has been established, the range of change of which can be divided into two segments that correspond to approximately equally probable values x_i . Simultaneously, the following table is arranged:

	$x_i < x_i^c$	$x_i > x_i^c$
$f < f_0$	A	B
$f > f_0$	C	D

where f stands for the value of the level of flood waters, f_0 stands for some average value that divides the range of flood waters change into intervals in which the values f are distributed approximately equally in number. A, B, C, D are the numbers of comparison results that correspond to the specified values f and x_i .

The following values are calculated:

$$A + B = n_1;$$

$$C + D = n_2;$$

$$A + C = n_3;$$

$$B + D = n_4.$$

Obviously, the total number of experiments is equal to either $n_1 + n_2$ or $n_3 + n_4$. The contingency ratio is calculated according to the formulas:

$$\psi = \frac{AD - BC}{\sqrt{(A+B)(C+D)(A+C)(B+D)}}. \quad (16)$$

If $\psi > 0,3$, then the relationship between the values is considered to be proved and significant; it should be studied in more detail, but if $\psi < 0,3$, then the relationship between the specified values can be considered insignificant.

Results

Methods of forecasting

The following method of estimating the level of flood waters and forecasting their development has been proposed. Suppose that by studying the floods that took place in the region under study with the help of methods (12) – (15) the ratio:

$$f_i = t^{n_i} \cdot e^{-a_i t}, \quad i = 1, \dots, k, \quad (17)$$

where k is the number of the studied floods that has been obtained.

The parameters x_1, x_2, \dots, x_m , that affect the formation and development of floods, are determined. They are water and physical properties of soils (water permeability, water intensity), air humidity, wind direction and speed, etc. It is considered that based on experimental studies, these values are known for each of the floods $x_1^i, x_2^i, \dots, x_m^i$. According to

the method (Pietrzak *et al.*, 2018) of associative analysis (16), for each of the values $x_s, s = 1, \dots, M$ the contingency ratio (16) is calculated and the level of connection between the corresponding x_s and f is determined. Thus, the number of values $x_i, i = 1, \dots, M$ that influence f , is reduced, and in the future only those $x_j, j = 1, \dots, M_0, M_0 \leq M$, that influence the process are considered. The presented results allow to optimize the experimental research procedure – the number of parameters $x_j, j = 1, \dots, M_0$, for which it is necessary to develop methods of experimental evaluation and control, is reduced. When studying the possibility of flooding in the given region, the values $x_j^p, j = 1, \dots, M_0$ are determined; after that the formula (17) and the parameters $x_j, j = 1, \dots, M_0$, determined for the floods f_i that influence the floods f_i , are found out.

The following value

$$\arg \min_i \left\{ \sum (x_j^i - x_j^p)^2 \right\} = j^*, \quad (18)$$

is determined in order to obtain more accurate forecast, the values $j_s^*, s = 1, \dots, N_0, N_0 (N_0 = 2)$ that are the nearest to j^* are determined. We choose those formulas (17) that correspond to the determined j_s^* . Applying the corresponding dependences f_i (17) determined for j_s^* (in the simplest case j_s^* is the only one and $j_s^* = j^*$, determined based on (18)), along with corresponding graphs, we choose the possible level of flooding and its intensity.

The scheme for estimating the level and duration of floods is estimated, for example, as shown in the graph (Figure 6). The values y_m and Δt are determined on the basis of statistical analysis.

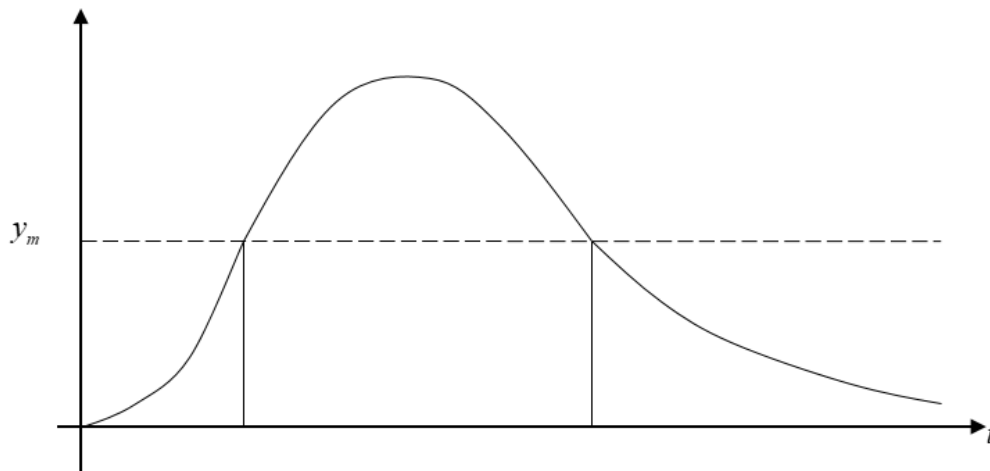


Figure 6: Scheme for estimating the flood level and duration [Explanation to figure 6: y_m stands for critical increase in the level of flood waters, Δt for duration of a flood.]

Conclusion

The method of estimating the flood water level and forecasting of flood phenomena has been suggested. Compared with the existing methods, the presented one is the method allows to offer the function analytical structure based on the statistical information about the floods in concrete region, which give the possibility to predict the flood's duration and intensity, to restore the ordinary differential equation which describe the flood process development using the information about one's solution. It is possible to change the order of differential equation to describe the processes more in detail, to apply the method of associative analysis for the choosing the factors which have the significant influence on the flood's characteristics. From the one side it is the method with the strict mathematical justification but from the second side it is quite simply in

realization in correspondent institution which activity is devoted to the struggle with the floods in our region. The direction of further research may be related to the development of experimental means to determine the parameters that affect the process of flood formation.

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Authors' Declarations and Essential Ethical Compliances

Authors' Contributions (in accordance with ICMJE criteria for authorship)

Contribution	Author 1	Author 2	Author 3	Author 4
Conceived and designed the research or analysis	Yes	Yes	No	No
Collected the data	Yes	Yes	Yes	No
Contributed to data analysis & interpretation	Yes	Yes	Yes	Yes
Wrote the article/paper	Yes	Yes	No	No
Critical revision of the article/paper	Yes	Yes	No	Yes
Editing of the article/paper	No	Yes	Yes	Yes
Supervision	No	Yes	No	Yes
Project Administration	Yes	No	No	No
Funding Acquisition	Yes	No	No	No
Overall Contribution Proportion (%)	33	33	14	20

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Research involving human bodies (Helsinki Declaration)

Has this research used human subjects for experimentation? No

Research involving animals (ARRIVE Checklist)

Has this research involved animal subjects for experimentation? No

Research involving Plants

The research did not involve plant species.

Research on Indigenous Peoples and/or Traditional Knowledge

Has this research involved Indigenous Peoples as participants or respondents? No

(Optional) PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses)

Have authors complied with PRISMA standards? Yes

Competing Interests/Conflict of Interest

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